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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2015

FIRST YEAR [BATCH 2015-18]

Date : 22/12/2015 **MATH FOR ECO [Gen]**

Time : 11 am – 2 pm **Paper : I** Full Marks : 75

[Use a separate Answer Book for each Group]

Group - A

(*Answer* ***any seven*** *questions*)

1. For any two sets A and B, prove that $(A\cup B)' = A'\cap B'$. [5]

2. Consider the mapping $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^3+1$. Check whether f is bijective? [5]

3. a) Let S = (0, 1] and $T=\left\{\frac{1}{n}: n\in\mathbb{N}\right\}$. Show that S – T is an open set. [3]

   b) Prove that the set $S=\{x\in\mathbb{R}: 2x^2-5x+2>0\}$ is open set. [2]

4. a) Prove that union of two open sets is open. [3]

   b) Give example of a boundary point which is not a limit point and also give a limit point which is not a boundary point. [2]

5. Prove that every convergent sequence is a bounded sequence. Is the converse true, explain why? (You can just provide a counter example if it is not true) [4+1]

6. If a sequence is monotone increasing and bounded above, then prove that it converges to it's supremum. [5]

7. Prove that the sequence $\{x_n\}$ where $x_n=\frac{3n+1}{n+2}$ is monotone increasing. Is it a convergent sequence? [5]

8. a) Prove that every convergent sequence is a Cauchy sequence. [3]

   b) Prove that $\left\{\frac{1}{n}\right\}$ is a Cauchy sequence. [2]

9. Use the Cauchy criterion of convergence of a series to investigate the convergence of $\sum_{n=1}^{\infty}\frac{1}{n!}$. [5]

10. Prove that the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is convergent. [5]

11. a) Prove that if $\sum_{n=1}^{\infty}a_n$ is convergent then $\underset{n\to\infty}{\text{Lt}}\, a_n = 0$. [3]

    b) Is the converse of the above statement true i.e. if $\underset{n\to\infty}{\text{Lt}}\, a_n = 0$ then whether $\sum_{n=1}^{\infty}a_n$ is convergent. [2]

12. State the Cauchy's Root test. Use it to prove that the series $\frac{1}{2}+\frac{1}{3^2}+\frac{1}{4^3}+\ldots$ is convergent. [2+3]

Group - B

(*Answer* ***any four*** *questions*)

13. a) Find the product of all the values of $(1+i\sqrt{3})^{3/4}$. [5]

    b) Prove that : $64\sin^5\theta\cos^2\theta=\sin7\theta-3\sin5\theta+\sin3\theta+5\sin\theta$. [5]

14. a) Consider the set of all mappings $\{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is bijective}\}$. Is the above set a group under composition of functions. [4]

b) Prove that the set $\mathbb{C}$ of all complex numbers form a field under addition and multiplication of complex numbers. [6]

15. a) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$. [5]

b) Show that $\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix}$ is divisible by λ^2 and find the other factor. [5]

16. a) Show that the matrix $A = \frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is orthogonal. Hence find A^{-1}. [5]

b) If $A = \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$, then show that $A^2 - 11A + 10I_3 = O$. Hence find A^{-1}. [5]

17. a) Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 4 & 6 & 2 \end{bmatrix}$. [5]

b) Find the fully reduced normal form of $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$. [5]

18. a) The national income equation is $u_{n+2} - 2au_{n+1} + au_n = I (0 < a < 1)$ assuming a and I constants, obtain u_n. [5]

b) The first term of a sequence is 1, the second term is 2, and every other term is the sum of the two preceding terms. Find the nth term. [5]

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